Modeling Dispersive Dielectrics for the 2-D TLM Method

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***Abstract*—The transition line matrix (TLM) method, since it is a time-domain method, cannot deal directly with dispersive media. We propose a way for modeling such media, starting from the causality relationship between field vectors D and E, which is discretized over the same space–time mesh as the two-dimensional TLM mesh. This leads to adding a voltage source at each node, its value depending on the previous instants. The method is validated by computing the reflection coefficient at an air–water interface and the cutoff frequencies of a rectangular waveguide containing a Debye dielectric.**

I. Introduction

IN time-domain methods, the behavior of frequency-dispersive dielectrics must be taken into account through the causality principle [1], which can be discretized in a two-dimensional (2-D) space–time mesh $(i\Delta x,\ j\Delta z,\ k\Delta t)$ (with $\Delta x = \Delta z$) as

$$_kD(i,j) = \varepsilon_0\left\{\varepsilon_\infty \cdot {_k E(i,j)} + \sum_{m=0}^{k-1} {_{k-m}E(i,j)} \cdot \chi_m\right\} \quad (1)$$

for each field component, ε_0 being the permittivity of the vacuum, ε_∞ the relative permittivity at very high frequencies, and

$$\chi_m = \int_{m\Delta t}^{(m+1)\Delta t} \chi(\tau) d\tau \quad (2)$$

$\chi(\tau)$ being the relative generalized susceptibility [2].

The aim of our method is to try to obtain a transition line matrix (TLM) equation that leads to an equation for the voltages similar to the discretized version of the field equation for the dispersive medium. Some models for the inclusion of frequency-dependent dielectrics in TLM have been developed recently [3]. Although the modelization leads to the same basic set of TLM equations than ours, our starting point is a wave equation, instead of computing the total charge at each node. Our approach uses an adaptation of the finite-difference time-domain (FDTD) technique, as exposed in [2], leading to a TLM equation, the main differences being the 2-D treatment and the starting point mentioned—not the usual Yee's scheme for the Maxwell equations. On the other hand, the computation of D requires a convolution that we compute at each time iteration, using the previous values. In [3] they propose to compute it for the cases when the generalized susceptibility leads to a differential equation in the time domain.

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To do so, we write down a "wave-like" equation that for the transverse component in TM modes in a 2-D mesh is

$$\mu_0 \frac{\partial^2 D}{\partial t^2} = \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial z^2}\,. \quad (3)$$

Discretizing it over the same mesh that (1) we can write

$$\begin{aligned} _{k+1}E_y(i,j) &= \frac{2}{4+Y_0} \\ &\times \Big\{ {_kE_y(i+1,j)} + {_kE_y(i-1,j)} \\ &\quad + {_kE_y(i,j+1)} + {_kE_y(i,j-1)} \\ &\quad + Y_0\, {_kE_y(i,j)} - \frac{4+Y_0}{2}\, {_{k-1}E_y(i,j)} \\ &\quad - 4\chi_0\, {_kE_y(i,j)} + 2\chi_0\, {_{k-1}E_y(i,j)} \\ &\quad - 2\sum_{n=0}^{k-1} {_{k-n}E_y(i,j)} \\ &\quad \times [\chi_{n+1} + \chi_{n-1} - 2\chi_n] \Big\} \end{aligned} \quad (4)$$

where $Y_0 = 4(\varepsilon_\infty + \chi_0) - 4$.

II. The TLM Method

For a 2-D TLM shunt network, loaded with stubs (for simulating a dielectric medium), the voltage in a node can be expressed as [4]

$$\begin{aligned} &_{k+1}V_y(i,j) \\ &= \frac{2}{4+Y_0}\{ {_{k+1}V_y^1(i,j)} + {_{k+1}V_y^2(i,j)} + {_{k+1}V_y^3(i,j)} \\ &\qquad + {_{k+1}V_y^4(i,j)} + Y_0\, {_{k+1}V_y^5(i,j)}\} \end{aligned} \quad (5)$$

where $Y_0 = 4(\varepsilon_r - 1)$.

We try to describe a dispersive medium by adding a source V_s (Fig. 1) of value $_{k+1}V_s(i,j)$ to the previous expression, so that when we obtain $_{k+1}V_y(i,j)$ in function of the previous V_y, we get an equation similar to (4). This leads to an equation for the voltage, depending on its previous values

$$\begin{aligned} &_{k+1}V_y(i,j) \\ &= \frac{2}{4+Y_0}\Big\{ {_kV_y(i,j+1)} + {_kV_y(i,j-1)} + {_kV_y(i+1,j)} \\ &\qquad + {_kV_y(i-1,j)} - \frac{4+Y_0}{2} \\ &\qquad \times {_{k-1}V_y(i,j)} + Y_0\, {_kV_y(i,j)}\Big\} \\ &\quad + {_{k+1}V_s(i,j)} - {_{k-1}V_s(i,j)}\,. \end{aligned} \quad (6)$$

TABLE I

CUTOFF FREQUENCIES (IN GHz) FOR A WAVEGUIDE 3 × 1.5 cm^2 FULL-FILLED/HALF-FILLED (FF/HF) WITH A DIELECTRIC OF $\varepsilon_\infty = 1$ AND $\varepsilon_S = 9$ AND $\tau_0 = 10^{-8}s$

	FF/HF	FF/HF	FF/HF	FF/HF	FF/HF	FF/HF	FF/HF
Analytic.	11.18/11.18	14.14/14.14	18.03/18.02	20.62/20.61	22.36/22.36	22.36/22.36	25.00/25.00
Commerc.	11.14/11.13	14.16/14.15	17.95/17.95	20.64/20.44	22.25/22.25	22.25/22.25	24.94/24.99
This met.	11.13/11.18	14.14/14.14	18.00/18.00	20.51/20.51	22.31/22.26	22.31/22.40	24.96/24.96

TABLE II

CUTOFF FREQUENCIES (IN GHz) FOR A WAVEGUIDE 3 × 1.5 cm^2 FULL-FILLED/HALF-FILLED (FF/HF) WITH A DIELECTRIC OF $\varepsilon_\infty = 1$ AND $\varepsilon_S = 9$ AND $\tau_0 = 10^{-11}s$

	FF/HF	FF/HF	FF/HF	FF/HF	FF/HF	FF/HF	FF/HF
Analytic.	3.76/9.94	4.78/10.20	6.14/12.29	7.06/14.55	7.70/14.71	7.70/16.92	8.67/17.33
Commerc.	3.77/9.87	4.84/9.87	6.10/12.38	6.99/14.53	7.71/14.53	7.71/16.86	8.61/17.40
This met.	3.68/9.50	4.71/10.79	6.19/12.52	7.85/14.00	7.85/14.00	7.85/16.94	9.52/16.94

TABLE III

CUTOFF FREQUENCIES (IN GHz) FOR A WAVEGUIDE 3 × 1.5 cm^2 FULL-FILLED/HALF-FILLED (FF/HF) WITH A DIELECTRIC OF $\varepsilon_\infty = 1$ AND $\varepsilon_S = 9$ AND $\tau_0 = 10^{-12}s$

	FF/HF	FF/HF	FF/HF	FF/HF	FF/HF	FF/HF	FF/HF
Analytic.	3.73/9.66	4.71/13.48	6.01/14.70	6.87/16.70	7.45/18.67	7.46/18.68	8.34/19.26
Commerc.	3.76/9.69	4.67/13.47	6.08/14.35	6.83/16.69	7.52/18.66	7.52/18.66	8.26/19.38
This met.	3.73/9.02	4.71/13.82	5.97/15.13	6.82/16.02	7.41/18.45	7.41/18.45	8.26/19.79

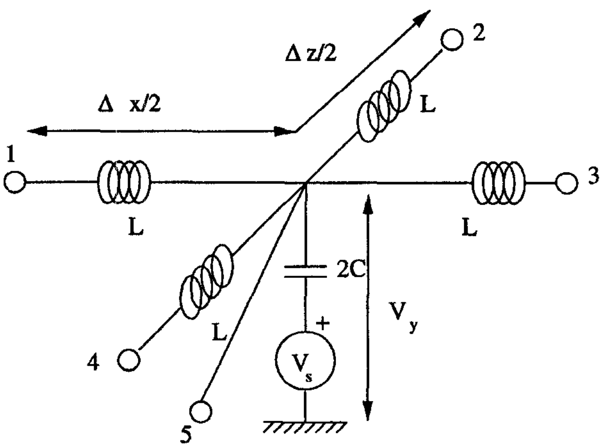

Fig. 1. Shunt node for 2-D TLM (including a source V_s).

So, if we compare (6) and (4), the value of $V_s(i,j)$ must satisfy

$$\begin{aligned} {}_{k+1}V_s(i,j) = {}& {}_{k-1}V_s(i,j) - \frac{4}{4+Y_0}\chi_0 \\ & \times [2_k V_y(i,j) - {}_{k-1}V_y(i,j)] \\ & - \frac{4}{4+Y_0}\sum_{n=0}^{k-1} {}_{k-n}V_y(i,j) \\ & \times (\chi_{n+1} + \chi_{n-1} - 2\chi_n) . \end{aligned} \quad (7)$$

In this way, any media can be modeled with a shunt network plus a stub of admittance Y_0 and a voltage source ${}_kV_s(i,j)$ verifying (7).

III. VALIDATION

For validating our model, we use a Debye dielectric whose time-domain generalized susceptibility is [2]

$$\chi_k = (\varepsilon_S - \varepsilon_\infty) \cdot e^{-k\Delta t/\tau_0} \cdot (1 - e^{-\Delta t/\tau_0}) \quad k \geq 0 \quad (8)$$

and zero otherwise. Here ε_S and ε_∞ stand for the low- and high-end frequency relative permittivities and τ_0 is the relaxation time. Therefore, the admittance Y_0 is

$$Y_0 = 2\varepsilon_\infty + 2(\varepsilon_S - \varepsilon_\infty) \cdot (1 - e^{\Delta t/\tau_0}) - 2 . \quad (9)$$

We have computed the cutoff frequencies of a rectangular waveguide, with 3 × 1.5 cm^2 section, divided in 26 × 13 cells, and performed 2^{11} time iterations. The frequencies are obtained through a FFT with Hanning window, to reduce leakage [5].

Two cases are considered: completely filled waveguide and half-filled waveguide, with the interface parallel to the narrow side. The values of ε_S and ε_∞ are nine and one, respectively. For τ_0 several values have been used.

The results are compared with those provided by analytical calculations and with those obtained with a commercially available TLM simulator [6]. In this case we have used monochromatic excitation, and for each frequency we have computed the complex permittivity and equivalent conductivity of the Debye dielectric. These are presented in the Tables I–III.

Using the same technique, we have computed the reflection coefficient at an air–water interface [2], [3] for a normal incident-plane wave. The domain is 600 (22.5 mm) × 10 (0.375 mm) square cells, for 4096 time steps, and the parameters of water are chosen as $\varepsilon_\infty = 1.8$, $\varepsilon_S = 81.0$, and $\tau_0 = 9.4 \times 10^{-12}$ s. Very good agreement with theoretical results are obtained. These are presented in Fig. 2.

IV. CONCLUSION

A 2-D model for dispersive dielectrics, suitable for use in the TLM method, has been presented. The results for a Debye dielectric are in good agreement with both analytical and other TLM simulations, for the cutoff frequencies of a waveguide

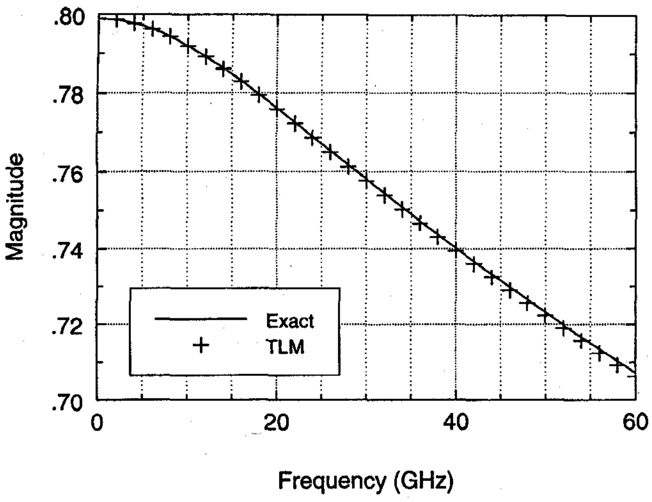

Fig. 2. Reflection coefficient at an air–water interface.

containing such dielectric, and for the reflection coefficient of an air–water interface. The differences observed between the commercial simulator and our method can be attributed to the need for longer calculations (i.e., more iterations in the computation of $V_s(i, j)$). On the other hand, the commercial simulator needs to be run once for each frequency. With this method a single simulation provide results over a range of frequencies.

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